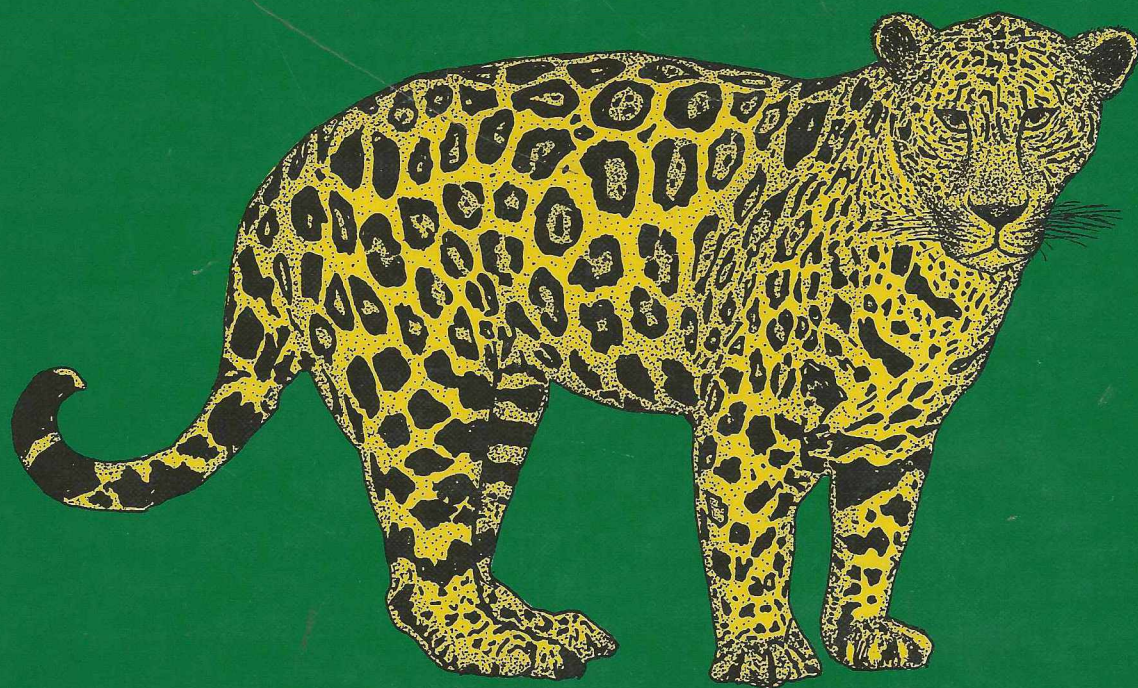


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Biomathematics Texts

J. D. Murray

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