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TEXTS AND MONOGRAPHS IN COMPUTER SCIENCE

FIRST-ORDER LOGIC AND AUTOMATED THEOREM PROVING

Melvin Fitting



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**First-Order Logic and
Automated Theorem Proving**

With 26 Illustrations



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